Fast fundamental domains for arithmetic Fuchsian groups in PARI/GP

James Rickards

CU Boulder

james.rickards@colorado.edu

5 August 2022

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- The code is written in PARI (a C library), and *not* in GP, so you will need to be running PARI/GP on a Linux kernel (e.g. Linux itself, a Linux server, Windows Subsystem for Linux, etc.)
- The package was compiled on the development version (2.14) of PARI/GP, so if you are running a stable version, you have to call "make" to build the appropriate library file.

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- Let $p \in \mathbb{H}$ have trivial stabilizer under the action of Γ. Then the space

$$D(p) := \{z \in \mathbb{H} : d(z, p) \leq d(gz, p) \text{ for all } g \in \Gamma\}$$

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- It is a connected region whose boundary is a closed hyperbolic polygon with finitely many sides, which come paired.

Example 1

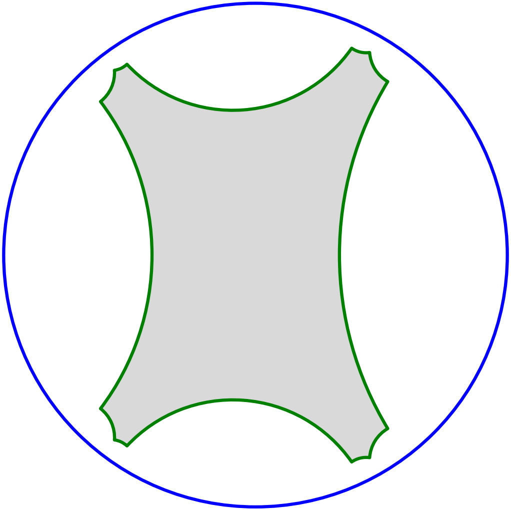

Figure 1: $F = \mathbb{Q}$, $\mathfrak{D} = 21$.

Example 2

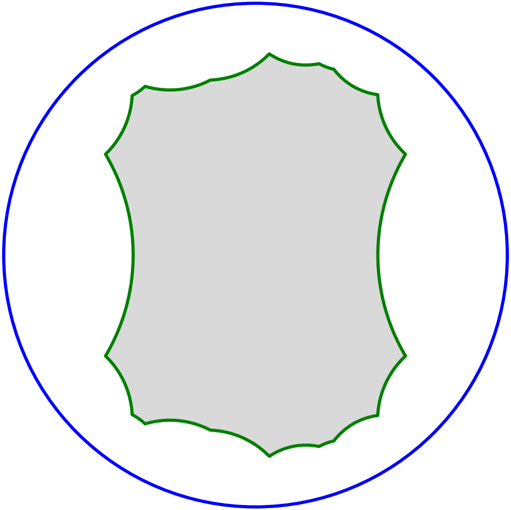

Figure 2: $F = \mathbb{Q}(\sqrt{5})$, $\mathrm{Nm}_{F/\mathbb{Q}}(\mathfrak{D}) = 61$.

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- Efficiently computing the intersection number of pairs of closed geodesics;
- And many more!

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- We will be focusing on computing Dirichlet domains for $\Gamma_{\mathcal{O}}$.

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?  I1=idealprimedec(F, 5)[1];
?  I2=idealprimedec(F, 17)[1]);
?  A2=alginit(F, [2, [[I1, I2], [1, 1]], [1, 1, 0]]);
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  4. If the area of the domain is $\mu(\Gamma_{\mathcal{O}})$, stop. Otherwise, go back to step 2.
- The running times were okay for small examples, but they did not scale well.

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- In 2015, Aurel Page generalized this algorithm to Kleinian groups ([Pag15]). His method to generate elements was probabilistic, and performed much better than Voight's method.

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- The Magma implementation for this is available from his website.

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- See [Ric21] for more details.

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- The constants cause the geometry to dominate for small areas, especially for n small.

Running times I

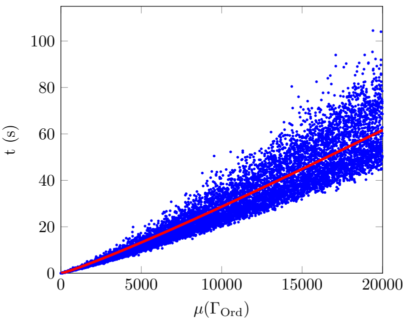

Figure 3: Time to compute the fundamental domain, $n = 1$.

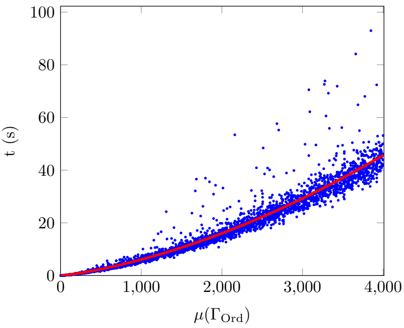

Figure 4: Time to compute the fundamental domain, $n = 2$.

Running times II

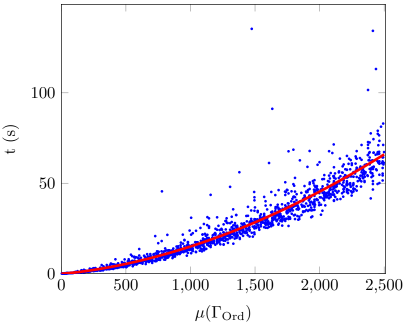

Figure 5: Time to compute the fundamental domain, $n = 3$.

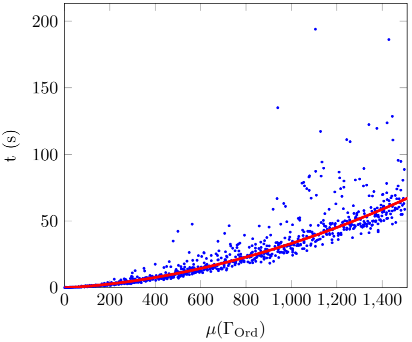

Figure 6: Time to compute the fundamental domain, $n = 4$.

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- `python_printfdom(U, "fdexample")`: prints the fundamental domain data into a file, ready to be viewed with python.

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- `python_printfdom(U, "fdexample")`: prints the fundamental domain data into a file, ready to be viewed with python.
- `fdom_latex(U, "fdom")`: writes the fundamental domain to a tex file.

Code in action

Since I can't embed gp in LaTeX, we will switch windows.

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- And more!

Acknowledgments and References

This research was supported by an NSERC Vanier Scholarship.

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